Unit - 2

logic

Statement for which we assign the truth values True/false are called atomic statement

Example: India is a country

(Connectives are also called logical operators)

Negation

If P is a statement then the negation of P is return as ~P or ¬P

P	~P
T	F
F	T

Conjunction

The conjunction of two statements P & Q is the statement P∧Q which is read as P & Q. The statement P∧Q has a truth table value when both P & Q is true otherwise it has truth value F.

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction:

The disjunction of two statement P & Q is the statement $P \vee Q$ which is read as P or Q

$P \vee Q$ has a truth table value F when the both P & Q is False otherwise it has truth table T

P	Q	P∨Q
T	T	T
T	F	T
F	T	T
F	F	F

Conditional and Biconditional

Conditional:

If P and Q are two statement then $P \to Q$ is read as P then Q is called conditional statement

$$\left(\begin{array}{l} P = T \\ Q = F \\ P \to Q = F \end{array} \right)$$

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional:

If P and Q are two statement the $P \rightleftarrows Q$ is read as P if and only if Q is called Biconditional statement.

P	Q	$P \rightleftarrows Q$
T	T	T
T	F	F
F	T	F
F	F	T

1. Construct the Truth table

a) $\sim(P \wedge Q)$ b) $\sim P \vee \sim Q$

a)

P	Q	$P \wedge Q$	$\sim(P \wedge Q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

b)

P	Q	$\sim P$	$\sim Q$	$\sim P \vee \sim Q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

2. Construct the Truth table

a) $P \wedge (P \rightarrow Q)$ b) $Q \wedge (P \vee \sim Q)$

a)

P	Q	$P \rightarrow Q$	$P \wedge (P \rightarrow Q)$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	T	F

b)

P	Q	$\sim Q$	$P \vee \sim Q$	$Q \wedge (P \vee \sim Q)$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	F
F	F	T	T	F

3. Construct the Truth table

a) $(Q \rightarrow P) \wedge (\sim P \wedge Q)$

a)

P	Q	$Q \rightarrow P$	$\sim P$	$\sim P \wedge Q$	$(Q \rightarrow P) \wedge (\sim P \wedge Q)$
T	T	T	F	F	F
T	F	F	F	F	F
F	T	F	T	T	F
F	F	T	T	F	F

Tautology:

A statement formula which is True regardless of the truth values of the statement which replace the variables is called universally valid formula or Tautology or logical Truth

Ex: $P \vee \sim P$

Contradiction:

A statement formula which is False regardless of the Truth values of the statement which replace the variables is called universally valid formula contradiction

Ex: $P \wedge \sim P$

1. Indicate which one are tautology or contradition

i) $(P \wedge Q) \rightleftarrows P$

P	Q	$P \wedge Q$	$(P \wedge Q) \rightleftarrows P$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

∴ It is not tautology or contradiction

ii) $P \rightarrow (P \vee Q)$

P	Q	$P \vee Q$	$P \rightarrow (P \vee Q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

∴ It is tautology

Equivalent Formula:

1) $P \vee P \Leftrightarrow P$

2) $(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$

3) $P \vee Q \Leftrightarrow Q \vee P$

4) $\sim (P \vee Q) \Leftrightarrow \sim P \wedge \sim Q$

5) $P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$

1. Prove the following implication $\sim(P \wedge Q) \Leftrightarrow \sim P \vee \sim$

L.H.S $\Rightarrow \sim (P \wedge Q)$

P	Q	$P \wedge Q$	$\sim(P \wedge Q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

R.H.S $\Rightarrow \sim P \vee \sim Q$

P	Q	$\sim P$	$\sim Q$	$\sim P \vee \sim Q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

L.H.S = R.H.S

$\sim (P \wedge Q) \Leftrightarrow \sim P \vee \sim Q$

$\therefore$ Hence proved

2. Prove the following equivalence.

ii) $\sim (P \rightarrow Q) \Leftrightarrow P \wedge \sim Q$

L.H.S $\Rightarrow \sim(P \to Q)$

P	Q	$P \to Q$	$\sim(P \to Q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

R.H.S $\Rightarrow P \wedge \sim Q$

P	Q	$\sim Q$	$P \wedge \sim Q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

L.H.S = R.H.S

$\sim(P \to Q) \Leftrightarrow P \wedge \sim Q$

$\therefore$ Hence proved

3. Show that P is equivalent to $\sim\sim P$, $P \wedge P$, $P \vee P$, $P \wedge (P \vee Q)$, $(P \wedge Q) \vee (P \wedge \sim Q)$ 10m

P	Q	$\sim P$	$\sim\sim P$	$P \wedge P$	$P \vee P$	$P \vee Q$	$P \wedge (P \vee Q)$	$P \wedge Q$
T	T	F	T	T	T	T	T	T
T	F	F	T	T	T	T	T	F
F	T	T	F	F	F	T	F	F
F	F	T	F	F	F	F	F	F
A			1	2	3		4	

$\sim Q$	$P \wedge \sim Q$	$(P \wedge Q) \vee (P \wedge \sim Q)$
F	F	T
T	T	T
F	F	F
T	F	F
		5

Hence $A \Leftrightarrow$ 1, 2, 3, 4, 5 is proved

* ~~Show that P is equivalent~~ to ¬¬P, P∧P, P∨P

Equivalent Form:

Let A & B be 2 statements formula. If the truth table value of A is equal to truth value of B then A & B is said to be equivalent.

Normal forms:

1. Principle disjunction normal form: (PDNF)
2. Principle conjunction Normal form: (PCNF)

PDNF:

A equivalent Formula consisting of disjunction of minterm only is known as Pdnf (or) sum-of-Product canonical form.

PCNF:

An equivalent formula consisting of disjunction of maxterms only is known as ~~conjunction~~ PCNF or product of sum canonical form.

1. obtain PCNF and PDNF

a) $Q \wedge (P \vee \sim Q)$

$\Rightarrow Q \wedge (P \vee \sim Q) \Rightarrow (Q \vee (P \wedge \sim P)) \wedge (P \vee \sim Q)$

$\Rightarrow (Q \vee P) \wedge (Q \vee \sim P) \wedge (P \vee \sim Q)$

$\Rightarrow$ Product of sum

$\Rightarrow$ PCNF.

b) $(Q \to P) \wedge (\sim P \wedge Q)$

$\Rightarrow (Q \to P) \wedge (\sim P \wedge Q)$

$\Rightarrow (\sim Q \vee P) \wedge (\sim P \wedge Q)$

$\Rightarrow (\sim Q \vee P) \wedge (\sim P \vee (Q \wedge \sim Q)) \wedge (Q \vee (P \wedge \sim P))$

$\Rightarrow (\sim Q \vee P) \wedge (\sim P \vee Q) \wedge (\sim P \vee \sim Q) \wedge (Q \vee P) \wedge (Q \vee \sim P)$

$\Rightarrow$ PCNF.

2. obtain the Pdnf Canonical form:

$(P \land Q) \lor (\sim P \land Q \land R)$

$(P \land Q) \lor (\sim P \land Q \land R)$

$\Rightarrow (P \land Q) \land (P \land \sim R) \lor (\sim P \land Q \land R)$

$\Rightarrow (P \land Q \land R) \lor (P \land Q \land \sim R) \lor (\sim P \land Q \land R)$

$\Rightarrow$ sum of product

$\Rightarrow$ pdnf.

3. Find the minterm, normal form (The principal disjunction normal form) $(\neg((P \lor Q) \land R)) \land (P \lor R))$

X $(\neg((P \lor Q) \land R)) \land (P \lor R))$

$\Rightarrow ((\neg P \land \neg Q) \lor \neg R) \land (P \lor R)$

$\Rightarrow ((\neg P \land \neg Q \land P) \lor (\neg P \land \neg Q \neg R) \lor (\neg R \land P) \lor (\neg R$

$\Rightarrow (\neg P \land P \land \neg Q) \lor (\neg P \land \neg Q \land R) \lor (\neg P \land P) \lor (\neg R \land R)$

$\Rightarrow (F \land \neg Q) \lor (\neg P \land \neg Q \land R) \lor (\neg R \land P) \lor F$

$\Rightarrow F \lor (\neg P \land \neg Q \land R) \lor (\neg R \land P) \lor F$

$\Rightarrow (\neg P \wedge \neg Q \wedge R) \vee (\neg R \wedge P)$

$\Rightarrow (\neg P \wedge \neg Q \wedge R) \vee (\neg R \wedge P \wedge (Q \vee \sim Q))$

$\Rightarrow (\neg P \wedge \neg Q \wedge R) \vee (\neg R \wedge P \wedge Q) \vee (\neg R \wedge P \wedge \sim Q)$

$\Rightarrow$ Pdnf

$\Rightarrow$ is the minterm normal form.

4. obtain Pcnf $(\sim P \rightarrow R) \wedge (Q \rightleftarrows P)$

$(\sim P \rightarrow R) \wedge (Q \rightleftarrows P)$

$\Rightarrow (P \vee R) \wedge (Q \rightarrow P) \vee (P \rightarrow Q)$

$\Rightarrow (P \vee R) \wedge (\sim Q \vee P) \vee (P \vee Q)$

$\Rightarrow (P \vee R) \vee (Q \wedge \sim Q) \wedge (\sim Q \vee P) \vee (R \wedge \sim R)$

$\wedge (\sim P \vee Q) \vee (R \wedge \sim R)$

$\Rightarrow (P \vee R \vee Q) \wedge (P \vee R \vee \sim Q) \wedge (\sim Q \vee P \vee R)$

$\wedge (\sim Q \vee P \vee \sim R) \wedge (\sim P \vee Q \vee R)$

$\wedge (\sim P \vee Q \vee \sim R)$

$\Rightarrow$ Pcnf